body of lepidopterous larvæ which adopt the methods of protective resemblance. Furthermore, it is very probable, as suggested by Professor Meldola, that the colour of the environment will prove to act as one of the determining causes of the larval colours ultimately assumed by the individuals of dimorphic species (which are generally green and brown in lepidopterous larvæ). To show in what a light this colour-relation appears to Dr. August Weismann (whose essay upon "The Markings of Caterpillars" first induced me to work at these organisms), I quote the following sentences from a letter I received from him after sending him my paper in the "Entomological Society's Transactions," Part I, April, 1884, in which this subject is alluded to :—

"Dagegen verstehe ich nicht ganz, wie sie sich den 'phytophagic character of the ground-colour' entstanden denken. Ich habe augenblicklich mein Buch nicht zur Hand u. kann deshalb die Note von Meldola nicht nachsehen, erinnere mich auch derselben nicht. Sie scheinen zu glauben, dass die *Nahrung* der Raupe bis zu einem gewissen Grad ihre Farbe *direkt* hervorrufe. Ich wäre sehr begierig, einen Beweis dafür kennen zu lernen. Ich kann mir nicht denken, wie dies möglich sein, solle jedoch weiche ich den *Thatsachen!* Ich bin begierig, zu erfahren, ob Sie solche inzwischen gefunden haben."

I venture to hope that the facts spoken of by Professor Weismann are now satisfactorily demonstrated, not as proving the former theory to which he alludes, that the food itself causes the change of colour after it has been eaten by the larva, but as proving the existence of the more subtle form of influence described in the present and in my last paper. At the same time I must express my sense of the great extent to which I am indebted to Professor Meldola, to whom we owe the former theory, for without the most suggestive editorial notes to his translation of Weismann's work, and the experiments undertaken by Mr. Boscher at his request, it is most improbable that the present investigation would ever have been begun.

III. "On the Polarisation of Light by Reflection from the Surface of a Crystal of Iceland Spar." By Sir John Conroy, Bart., M.A., of Keble College, Oxford. Communicated by Professor G. G. Stokes, P.R.S. Received January 27, 1886.

(Plate 2.)

In the year 1819 Sir David Brewster communicated to the Royal Society ("Phil. Trans.," 1819, p. 145) an account of some experiments he had made on the polarisation of light by reflection from the surface

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of double refracting substances, and showed that Malus' statement with regard to Iceland spar was incorrect.

Malus said that Iceland spar behaves towards the light it reflects like a common transparent body, and that its polarising angle is about 56° 30′, and that whatever be the angle comprehended between the plane of incidence and the principal section of the crystal, the ray reflected by the first surface is always polarised in the same manner ("Théorie de la Double Refraction," pp. 240, 241).

Some years later Seebeck ("Pogg. Ann.," vol. xxi, p. 290; vol. xxii, p. 196; vol. xxxviii, p. 276; vol. xl, p. 462) made a number of very accurate observations on the same subject, and in 1835 and 1837 Neumann published in "Pogg. Ann.," vol. xl, 497, and vol. xlii, p. 1, an account of further experiments that he had made on the reflection of light by Iceland spar.

He begins his second paper by a brief summary of the results obtained by Brewster and Seebeck. "Brewster found that the angle of complete polarisation for calcspar depends on the position of the reflecting surface relatively to the axis, and upon the position of its principal section to the plane of reflection; he also found that when the reflecting surface is covered with a liquid, the plane of polarisation of the completely polarised ray does not coincide with the plane of reflection, but makes a smaller, or greater, angle with this; when a cleavage-face of calcspar is covered with oil of cassia this deviation may amount to 90°. The knowledge of these phenomena has only been further advanced in recent times. Dr. Seebeck has so followed out, by means of most accurate determinations, the influence of optically uniaxial crystals upon complete polarisation, that the angle of incidence at which this occurs can be determined as accurately beforehand as it can by Brewster's law in the case of uncrystallised bodies. Seebeck also discovered that the deviation of the plane of polarisation from the plane of reflection, which Brewster had observed, also occurs when the ray of light falls directly from air on to the surface of the crystal."

Seebeck's observations having been mainly directed to the determination of the angle of polarisation, Neumann's object was to determine the azimuth of the plane of polarisation of the reflected light. They both assumed, contrary to Fresnel's hypothesis, that the density of the ether in the two media was the same and the elasticity different, and therefore that the plane of vibration coincided with the plane of polarisation, and starting with this assumption succeeded in showing that the observed and calculated results were in close accordance.

Seebeck and Neumann only repeated a portion of Brewster's experiments, and no one except Sir David Brewster appears to have made any determinations of the angles and azimuths of polarisation when the spar was in contact with media other than air.

Professor Stokes very kindly called my attention to these experiments of Sir David Brewster, and pointed out that as they had never been published in detail, and had not been repeated by anyone else, it was desirable that further observations should be made on this subject. The experiments, the results of which I have the honour of submitting to the Royal Society, were undertaken at Professor Stokes' suggestion, and in carrying them out I had the benefit of his advice.

The apparatus used was essentially the same as that employed by Seebeck; the divided circle of the goniometer was, however, horizontal, and not vertical, as in Seebeck's instrument, and the arrangement for keeping the reflected ray constantly in the axis of the observing tube, whilst the angle of incidence was varied, differed from that employed by him; the axes of the stage and of the observing tube were fitted with toothed wheels which geared into a double pinion, the diameters of the wheels being such that the angular velocity of the observing tube was double that of the stage.

The goniometer had a vertical stage in addition to the ordinary horizontal one, which could be moved nearer to and further from the axis of the instrument, and this stage had four adjusting screws, so that the front surface could be placed parallel to the axis of rotation.

A brass plate was clamped to this stage; a short brass tube carrying an annular toothed wheel at one end, and a divided collar at the other, fitted into an aperture at the lower end of this plate, and could be rotated in the plane of the plate by turning a milled head fixed at the end of a rod, which carried a bevel pinion working into the annular wheel.

A brass tube with a collar at one end could be fastened to the annular wheel by four screws passing through holes in the collar, the back of the collar and the inner surface of the wheel being portions of a spherical surface.

The crystal whose reflective power was to be examined was fixed in the inner tube, which was then adjusted by means of the four screws so that the surface of the crystal was in the plane of rotation, and then, by altering the position of the vertical stage, the plane of rotation brought vertically over the axis of the goniometer.

The crystal was cemented into the tube with plaster of Paris; a little lard was rubbed on the edges of the face which was to be exposed, and it was placed on a plate of glass with this surface downwards, and the brass tube, the collar of which had also been greased, placed round it, and centred by means of a marked card placed under the glass, and plaster of Paris poured in. In one of the earlier experiments the crystal was found after a certain number of observations had been made to have become loose, owing to the plaster having shrunk away from the tube; three holes about 2 mm. in diameter were therefore drilled in the sides of the tube, and no further difficulty was

experienced from this cause, as the little projections of plaster which filled these holes effectually prevented any movement.

In order to adjust the reflecting surface, a diaphragm with a small hole in it was fitted into the eye end of the observing tube, and a lamp with a flat wick so placed that its image was seen by reflection from the surface of the crystal, two of the adjusting screws of the tube being in the horizontal plane (*i.e.*, the plane of incidence). The tube was then turned through an angle of 180°, and the reflected image brought back half way into the centre of the field by altering the two screws of the tube which were in the plane of incidence, and the remainder of the distance by means of the screws of the stage, which, as well as the observing tube, remained clamped to the horizontal circle whilst this adjustment was being made.

The tube was then turned back to its original position, and the adjustment repeated if necessary; the tube was then turned through 90°, and the second pair of screws altered till the reflected image remained in the centre of the field whilst the crystal was rotated. The adjustment was then examined by means of a simple form of diagonal eye-piece placed in the collimator tube of the goniometer, consisting of a brass tube with a diaphragm at either end, with a small aperture in each, and also in the side of the tube; inside the tube, and opposite the aperture in the side, a piece of microscopical cover-glass was fixed at an angle of 45° with the axis of the tube; the lamp was placed opposite the aperture in the side of the tube, and the vertical stage rotated until the light of the lamp reflected from the thin glass was reflected back by the crystal along the axis of the eye-piece to the observer; the tube holding the crystal was then rotated, and if the spot of light remained visible whilst the tube made a complete rotation, the adjustment was considered to have been correctly made, and the position of the stage was then read on the horizontal circle of the goniometer, and this measurement taken as that of perpendicular incidence.

Several such readings were made, and then the position of the tube and lamp altered, and several more readings made, and the mean of these, which usually were close together, taken as the zero for the angle of incidence.

Two complete series of observations were made with cleavage-faces of Iceland spar in air, water, and tetrachloride of carbon, the water and tetrachloride of carbon being contained by a nearly cylindrical thin glass vessel (a chemical beaker), which stood on the horizontal stage of the goniometer, the tetrachloride being prevented from evaporating by a layer of water floating on its surface.

When the reflection took place in air, a paraffin lamp, with a flat burner placed edgeways (*i.e.*, radially to the goniometer) was used as the source of light; when the crystal was in water or tetrachloride

of carbon, its reflecting power was so much diminished that a more intense source of light was necessary, and a magic lantern (a "sciopticon") was used, a black card with a slit 3·5 mm. wide being placed in the slide-holder, and focussed on the surface of the crystal, care being taken in both cases that the direction of the incident light should coincide as nearly as possible with the axis of the collimator tube.

The measurements were made by altering the angle of incidence and the azimuth of the observing Nicol until the light was reduced to a minimum, the position of the crystal remaining fixed.

In order to obtain anything like accurate results with observations of this kind it is necessary to make a large number of determinations and take their mean: it was obvious that there were two ways in which any given number of observations might be grouped, either by making a good many separate determinations for a few positions of the crystal, or by making a few observations at a number of different azimuths; the latter alternative being the one adopted, two readings were made at seventy-two different azimuths of the crystal.

In the first series the observations started from one of the edges of the crystal, the tube containing it being turned through 10° after each pair of readings; after thirty-six pairs of readings the crystal was turned through 6° 20′, and then thirty-six more double readings made at intervals of 10° from each other.

In the second series the observations started from the principal section, and were also made at intervals of 10°; the crystal was turned through 5° after thirty-six observations had been made, and then thirty-six more were made, also at intervals of 10°.

It was thought that by working in this way the results would be more independent of each other, and therefore more trustworthy than if the readings had been made continuously round the whole circle. The position of the crystal was determined by placing a square on the horizontal stage of the goniometer, and rotating the tube carrying the crystal until the edge of the crystal appeared to coincide with the vertical edge of the square, and noting the reading of the divided ring attached to the tube; several such readings were made, and the tube and crystal turned through 180° and several more observations made, and the mean of these taken as the position in which one of the sides of the crystal was vertical (*i.e.*, perpendicular to the plane of incidence); the position of the principal section could then be readily determined as it bisects the obtuse angle, and therefore, that angle being one of 101° 55′, or nearly 102°, forms an angle of about 51° with the adjacent edges.

The position of the crystal in which the principal section was in the plane of incidence and the obtuse summit nearest the observer was considered the zero position; when the principal section was in

the plane of incidence and the obtuse summit towards the side from which the light was incident upon it, was therefore azimuth 180°. The crystal was rotated clockwise, and the same direction of rotation was considered the positive direction for the Nicol.

Table I gives the measurements made in this way with a cleavage-face of Iceland spar in air; Tables II and III with the same face in water and carbon tetrachloride. Table IV contains the measurements made with another cleavage-face of the same crystal in water. Tables V, VI, and VII, give the results with a cleavage-face of a second crystal in air, water, and carbon tetrachloride.

Table I.

Iceland Spar in Air.

Azimuth of the principal section of the crystal.	Azimuth of the plane of polarisation.	Angle of polarisation.	Azimuth of the principal section of the crystal.	Azimuth of the plane of polarisation.	Angle of polarisation.	Difference in the values of the polarising angle at θ and $\theta+180°$.
° ′	° ′	° ′	° ′	° ′	° ′	′
+1 0	+0 10	57 20	181 0	+0 22	57 06	+14
7 20	+0 35	57 24	187 20	+0 47	57 09	+15
11 0	+0 30	57 17	191 0	+1 10	57 12	+ 5
17 20	+0 40	57 22	197 20	+1 50	57 16	+ 6
21 0	+0 25	57 24	201 0	+2 05	57 25	− 1
27 20	+0 45	58 0	207 20	+2 35	57 29	+31
31 0	+0 47	57 50	211 0	+3 15	57 59	− 9
37 20	+0 37	58 11	217 20	+3 25	57 59	+12
41 0	+0 40	58 17	221 0	+3 30	58 19	− 2
47 20	+0 47	58 39	227 20	+3 37	58 31	+ 8
51 0	+0 35	58 34	231 0	+4 05	58 34	0
57 20	+0 35	59 02	237 20	+3 42	58 52	+10
61 0	−0 03	58 48	241 0	+3 45	59 03	−15
67 20	−0 10	59 18	247 20	+3 32	59 12	+ 6
71 0	+0 02	59 13	251 0	+3 37	59 15	− 2
77 20	−1 08	59 32	257 20	+3 50	59 42	−10
81 0	−1 25	59 24	261 0	+3 05	59 32	− 6
87 20	−1 43	59 35	267 20	+3 10	59 20	+15
91 0	−1 33	59 49	271 0	+2 45	59 38	+11
97 20	−2 30	59 26	277 20	+2 12	59 33	− 7
101 0	−2 30	59 38	281 0	+2 35	59 45	− 7
107 20	−2 33	59 25	287 20	+1 15	59 23	+ 2
111 0	−2 23	59 11	291 0	+1 47	59 20	− 9
117 20	−2 48	59 06	297 20	+0 52	59 03	+ 3
121 0	−2 58	59 04	301 0	+0 55	58 56	+ 8
127 20	−3 13	58 43	307 20	+0 30	58 43	0
131 0	−3 05	58 37	311 0	+0 05	58 37	0
137 20	−3 0	58 25	317 20	−0 20	58 25	0
141 0	−2 55	58 10	321 0	0	58 17	− 7
147 20	−2 37	57 47	327 20	+0 07	58 06	−19
151 0	−2 20	57 54	331 0	−0 03	57 48	+ 6
157 20	−2 08	57 26	337 20	−0 08	57 54	−28
161 0	−1 33	57 43	341 0	−0 25	57 30	+13
167 20	−0 48	57 22	347 20	−0 06	57 22	0
171 0	−1 0	57 28	351 0	+0 05	57 23	+ 5
177 20	+0 12	57 14	357 20	−0 13	57 27	−13

Table II.

Iceland Spar in Water.

Azimuth of the principal section of the crystal.	Azimuth of the plane of polarisation.	Angle of polarisation.	Azimuth of the principal section of the crystal.	Azimuth of the plane of polarisation.	Angle of polarisation.	Difference in the values of the polarising angle at θ and $\theta+180°$.
+1° 0′	+0° 12′	49° 19′	181° 0′	−0° 03′	49° 18′	+1′
7 20	+0 40	50 29	187 20	+2 0	49 33	+56
11 0	+1 22	49 42	191 0	+3 02	49 11	+31
17 20	+1 55	49 59	197 20	+5 17	49 26	+33
21 0	+1 10	50 10	201 0	+6 37	49 54	+16
27 20	+1 12	51 15	207 20	+7 55	51 07	+8
31 0	+0 25	50 30	211 0	+7 55	50 06	+24
37 20	+1 27	51 30	217 20	+10 27	50 44	+46
41 0	+0 42	51 33	221 0	+11 22	51 14	+19
47 20	+0 22	52 30	227 20	+12 27	52 28	+2
51 0	−0 23	52 21	231 0	+13 07	52 07	+14
57 20	−2 20	53 37	237 20	+13 37	53 24	+13
61 0	−2 48	53 52	241 0	+13 42	53 53	−1
67 20	−3 03	54 23	247 20	+13 47	54 50	−27
71 0	−5 38	54 49	251 0	+13 42	54 52	−3
77 20	−8 23	55 56	257 20	+13 40	55 14	+42
81 0	−7 35	55 32	261 0	+12 52	55 38	−6
87 20	−8 40	55 32	267 20	+12 10	56 07	−35
91 0	−9 33	55 37	271 0	+11 05	56 11	−34
97 20	−10 55	56 05	277 20	+10 15	55 47	+18
101 0	−12 08	56 08	281 0	+9 20	55 49	+19
107 20	−13 25	55 34	287 20	+7 0	55 41	−7
111 0	−12 38	54 58	291 0	+7 10	55 34	+24
117 20	−14 03	54 07	297 20	+4 57	54 13	−6
121 0	−13 0	54 18	301 0	+3 10	53 23	+55
127 20	−12 24	52 41	307 20	+1 45	53 44	−63
131 0	−12 13	53 04	311 0	+1 52	53 42	−38
137 20	−12 0	53 02	317 20	−0 10	52 23	+39
141 0	−11 0	51 09	321 0	+1 05	51 50	−41
147 20	−9 10	51 04	327 20	+0 10	51 42	−38
151 0	−8 58	49 52	331 0	+0 37	51 12	−80
157 20	−6 38	49 42	337 20	+0 17	50 39	−57
161 0	−6 10	48 46	341 0	−0 25	49 53	−67
167 20	−4 13	50 20	347 20	+0 27	49 05	+75
171 0	−2 45	49 18	351 0	+0 45	49 55	−37
177 20	−1 55	49 10	357 20	−0 15	49 25	−15

Table III.

Iceland Spar in Tetrachloride of Carbon.

Azimuth of the principal section of the crystal.	Azimuth of the plane of polarisation.	Angle of polarisation.	Azimuth of the principal section of the crystal.	Azimuth of the plane of polarisation.	Angle of polarisation.	Difference in the values of the polarising angle at θ and $\theta+180°$.
° ′	° ′	° ′	° ′	° ′	° ′	′
+1 0	+ 0 22	44 34	181 0	− 0 43	46 39	−125
7 20	− 0 18	46 35	187 20	+ 4 37	45 15	+ 80
11 0	+ 0 17	45 46	191 0	+ 5 45	44 20	+ 86
17 20	+ 0 42	45 40	197 20	+ 7 12	45 19	+ 21
21 0	+ 1 05	46 22	201 0	+11 35	45 18	+ 64
27 20	+ 1 37	47 43	207 20	+14 02	46 55	+ 48
31 0	+ 1 55	47 0	211 0	+16 55	47 39	− 39
37 20	+ 0 07	48 21	217 20	+19 15	48 02	+ 19
41 0	+ 0 07	49 34	221 0	+22 40	50 47	− 73
47 20	− 2 38	50 05	227 20	+24 50	51 0	− 55
51 0	− 0 38	50 36	231 0	+26 30	53 03	−147
57 20	− 4 50	53 37	237 20	+27 40	53 51	− 14
61 0	− 7 25	55 39	241 0	+31 12	57 48	−129
67 20	− 8 48	56 10	247 20	+31 22	58 15	−125
71 0	−16 50	60 37	251 0	+33 47	61 19	− 42
77 20	−19 20	59 41	257 20	+32 47	62 35	−174
81 0	−23 10	61 27	261 0	+33 25	64 59	−212
87 20	−26 25	62 40	267 20	+32 20	64 43	−123
91 0	−30 23	64 47	271 0	+31 17	64 11	+ 36
97 20	−28 20	60 50	277 20	+28 02	64 16	−206
101 0	−31 45	64 09	281 0	+26 30	64 49	− 40
107 20	−32 15	63 0	287 20	+20 57	62 02	+ 58
111 0	−32 30	63 15	291 0	+21 37	63 11	+ 4
117 20	−28 38	58 43	297 20	+15 02	57 45	+ 58
121 0	−32 0	62 39	301 0	+13 52	58 27	+252
127 20	−27 45	55 34	307 20	+10 40	58 48	−194
131 0	−29 40	58 34	311 0	+ 5 42	55 47	+167
137 20	−22 13	55 23	317 20	+ 2 50	50 25	+298
141 0	−22 25	52 12	321 0	+ 1 15	52 31	− 19
147 20	−20 35	52 30	327 20	+ 1 25	49 41	+169
151 0	−17 25	51 06	331 0	+ 1 02	49 49	+ 77
157 20	−12 50	44 49	337 20	− 0 10	46 58	−129
161 0	−13 35	45 39	341 0	+ 0 30	48 49	−190
167 20	− 6 35	45 04	347 20	− 0 15	46 22	− 78
171 0	− 5 30	46 12	351 0	− 0 08	47 31	− 79
177 20	− 2 35	44 08	357 20	− 0 53	45 56	−108

Table IV.

Iceland Spar in Water.

Azimuth of the principal section of the crystal.	Azimuth of the plane of polarisation.	Angle of polarisation.	Azimuth of the principal section of the crystal.	Azimuth of the plane of polarisation.	Angle of polarisation.	Difference in the values of the polarising angle at θ and $\theta+180°$.
°	° ′	° ′	°	° ′	° ′	′
+ 1	+ 1 22	48 42	181	+ 0 37	49 43	−61
11	+ 0 50	49 10	191	+ 4 02	49 25	−15
21	− 0 30	48 43	201	+ 7 17	49 52	−69
31	+ 1 25	50 11	211	+ 9 45	50 35	−24
41	− 0 18	50 46	221	+11 35	51 30	+16
51	− 0 23	52 19	231	+12 47	52 59	−40
61	− 2 43	53 49	241	+13 07	53 45	+ 4
71	− 5 43	54 48	251	+13 42	55 11	−23
81	− 8 0	55 23	261	+11 50	55 16	+ 7
91	−10 28	55 36	271	+11 01	55 24	+12
101	−12 30	55 05	281	+ 8 25	55 25	−20
111	−14 40	54 31	291	+ 5 22	54 20	+11
121	−12 40	53 43	301	+ 3 20	53 29	+14
131	−12 23	51 30	311	+ 2 0	52 10	−40
141	−12 0	51 33	321	+ 1 37	51 48	−15
151	− 7 23	50 15	331	− 1 0	51 15	−60
161	− 3 48	49 35	341	− 0 48	48 48	+47
171	− 1 23	49 19	351	+ 0 22	49 11	+ 8

Table V.

Iceland Spar in Air.

Azimuth of the principal section of the crystal.	Azimuth of the plane of polarisation.	Angle of polarisation.	Azimuth of the principal section of the crystal.	Azimuth of the plane of polarisation.	Angle of polarisation.	Difference in the values of the polarising angle at θ and $\theta+180°$.
+0°	+0° 32′	57° 24′	180°	+0° 37′	56° 46′	+38′
5	+0 45	56 56	185	+1 02	56 22	+34
10	+0 40	57 29	190	+1 20	56 52	+37
15	+0 57	56 57	195	+0 55	56 39	+18
20	+0 57	57 34	200	+2 17	57 10	+24
25	+1 0	57 11	205	+2 57	56 47	+24
30	+0 52	57 54	210	+3 02	57 36	+18
35	+1 12	57 37	215	+3 50	57 11	+26
40	+1 0	58 11	220	+3 35	57 56	+15
45	+0 47	58 03	225	+4 02	57 51	+12
50	+0 22	58 50	230	+3 45	58 10	+40
55	+0 30	58 28	235	+3 57	58 06	+22
60	+0 20	59 19	240	+3 25	58 36	+43
65	0	58 59	245	+4 02	59 19	−20
70	−0 50	59 33	250	+3 30	59 19	+14
75	−0 38	59 04	255	+3 30	58 52	+12
80	−1 05	59 49	260	+3 07	59 15	+34
85	−1 23	59 22	265	+3 20	59 03	+19
90	−1 50	60 07	270	+2 17	59 39	+28
95	−2 05	59 24	275	+2 50	59 0	+24
100	−1 50	59 43	280	+1 42	59 37	+ 6
105	−2 53	59 31	285	+1 42	58 50	+41
110	−2 55	59 36	290	+0 50	59 23	+13
115	−2 53	58 56	295	+0 55	58 50	+ 6
120	−2 58	59 04	300	+0 42	58 34	+30
125	−3 03	58 47	305	+0 17	58 27	+20
130	−3 05	58 22	310	+0 07	58 47	−25
135	−2 55	58 27	315	+0 15	58 01	−34
140	−2 28	57 59	320	−0 15	58 06	− 7
145	−2 23	57 13	325	−0 05	58 37	−80
150	−2 55	57 11	330	−0 25	57 54	−43
155	−1 35	56 46	335	−0 03	57 17	−31
160	−1 20	57 21	340	−0 25	57 21	0
165	−0 43	56 24	345	+0 02	57 05	−41
170	−0 28	56 56	350	0	57 28	−32
175	+0 10	56 22	355	+0 22	56 58	−36

Table VI.

Iceland Spar in Water.

Azimuth of the principal section of the crystal.	Azimuth of the plane of polarisation.	Angle of polarisation.	Azimuth of the principal section of the crystal.	Azimuth of the plane of polarisation.	Angle of polarisation.	Difference in the values of the polarising angle at θ and $\theta+180°$.
+ 0°	− 0° 08′	48° 43′	180°	+ 0° 35′	48° 25′	+18′
5	+ 0 17	48 38	185	+ 1 32	48 17	+21
10	+ 0 32	49 04	190	+ 3 0	48 39	+25
15	+ 1 45	48 56	195	+ 5 35	48 43	+13
20	+ 0 50	49 09	200	+ 5 47	49 27	−18
25	+ 1 15	49 32	205	+ 7 47	49 15	+17
30	+ 0 47	50 14	210	+ 9 07	49 47	+27
35	+ 0 55	49 53	215	+ 9 27	50 10	−17
40	+ 0 12	51 14	220	+ 9 40	50 46	+28
45	− 0 13	51 14	225	+11 50	50 57	+17
50	− 0 20	51 33	230	+12 50	52 09	−36
55	− 1 15	51 52	235	+12 12	52 19	−27
60	− 2 15	52 38	240	+13 0	53 05	−27
65	− 3 25	53 43	245	+14 27	53 23	+20
70	− 4 38	53 50	250	+13 15	54 08	−18
75	− 5 45	54 32	255	+12 30	54 28	+ 4
80	− 7 50	54 45	260	+12 07	55 08	−23
85	− 7 13	54 18	265	+10 57	54 55	−37
90	−10 23	55 19	270	+10 32	55 20	− 1
95	− 9 55	54 25	275	+ 9 30	55 34	−69
100	−10 58	54 22	280	+ 9 40	55 40	−78
105	−11 15	53 35	285	+ 7 22	54 45	−70
110	−11 45	54 15	290	+ 6 15	54 07	+ 8
115	−10 58	53 35	295	+ 5 15	54 12	−37
120	−11 43	53 05	300	+ 3 40	52 39	+26
125	−10 43	52 15	305	+ 2 27	52 45	−30
130	−10 58	51 23	310	+ 1 22	52 05	−42
135	−10 55	51 25	315	+ 1 37	52 16	−51
140	−10 08	51 07	320	− 0 30	51 25	−18
145	− 9 0	49 54	325	− 0 33	50 29	−35
150	− 8 48	50 01	330	− 0 13	50 07	− 6
155	− 6 15	49 36	335	+ 0 07	49 55	−19
160	− 5 13	49 13	340	− 0 05	49 26	−13
165	− 3 48	49 02	345	+ 0 10	49 0	+ 2
170	− 2 13	48 38	350	+ 0 15	48 59	−21
175	− 1 0	48 43	355	− 0 05	48 30	+13

Table VII.

Iceland Spar in Tetrachloride of Carbon.

Azimuth of the principal section of the crystal.	Azimuth of the plane of polarisation.	Angle of polarisation.	Azimuth of the principal section of the crystal.	Azimuth of the plane of polarisation.	Angle of polarisation.	Difference in the values of the polarising angle at θ and $\theta+180^{\circ}$.
$+\overset{\circ}{0}$	$-\overset{\circ}{0}\ \overset{\prime}{10}$	$\overset{\circ}{45}\ \overset{\prime}{07}$	$+\overset{\circ}{180}$	$+\overset{\circ}{0}\ \overset{\prime}{30}$	$\overset{\circ}{44}\ \overset{\prime}{44}$	$+\overset{\prime}{23}$
5	+ 0 35	45 28	185	+ 2 32	45 05	+ 23
10	+ 0 45	45 55	190	+ 5 17	45 47	+ 8
15	+ 0 55	46 52	195	+ 8 27	45 41	+ 71
20	+ 2 22	46 36	200	+11 45	46 30	+ 6
25	+ 1 20	47 06	205	+14 42	47 26	− 20
30	+ 0 17	48 06	210	+15 57	47 45	+ 21
35	− 0 10	48 38	215	+17 17	48 13	+ 25
40	− 0 23	49 41	220	+21 57	50 17	− 36
45	− 2 0	50 37	225	+24 52	52 17	−100
50	− 3 25	52 14	230	+26 07	53 19	− 65
55	− 5 58	53 41	235	+26 45	54 12	− 31
60	− 6 55	54 30	240	+31 20	57 26	−176
65	−14 43	58 23	245	+33 07	58 32	− 9
70	−15 45	60 08	250	+30 45	59 48	+ 20
75	−20 55	60 18	255	+29 30	59 31	+ 47
80	−22 28	60 43	260	+34 15	65 43	−300
85	−26 25	62 13	265	+32 32	63 24	− 71
90	−25 28	59 42	270	+29 05	63 18	−216
95	−29 42	61 05	275	+25 52	61 52	− 47
100	−31 10	61 58	280	+26 47	64 47	−169
105	−32 13	61 24	285	+22 07	60 32	+ 52
110	−28 13	58 07	290	+17 15	58 57	− 50
115	−28 08	56 47	295	+15 42	58 07	− 80
120	−30 50	57 17	300	+10 32	57 04	+ 13
125	−27 50	56 13	305	+ 9 47	55 47	+ 26
130	−22 53	51 59	310	+ 7 05	54 09	−130
135	−21 30	51 04	315	+ 3 32	51 09	− 5
140	−21 40	49 23	320	+ 1 42	50 38	− 75
145	−18 58	49 15	325	+ 0 25	48 40	+ 35
150	−15 15	47 08	330	− 0 05	48 12	− 64
155	−12 35	47 30	335	− 1 03	47 11	+ 19
160	−10 43	46 15	340	− 0 05	47 25	− 70
165	− 8 18	46 12	345	− 0 10	45 48	+ 24
170	− 4 58	45 16	350	+ 0 05	46 02	− 46
175	− 2 30	45 11	355	+ 0 10	45 06	+ 5

It had been intended to make similar measurements with artificial surfaces cut perpendicular and parallel to the axis of the crystal, and three pieces of Iceland spar cut respectively parallel to a natural face, and perpendicular and parallel to the axis, and all polished with "whiting" were obtained.

Seebeck states ("Pogg. Ann.," vol. xxi, 290) that Iceland spar polished with rouge or putty powder differs in its optical properties

from the natural substance, but that an artificial surface polished with chalk behaves very nearly, if not exactly, like a natural one.

Seebeck's measurements were all made with the crystal in air, and as the changes in the azimuth of the plane of polarisation, and in the value of the polarising angle, for different azimuths of the crystal, when such is the case are small, it seemed desirable before making any measurements with the artificial surfaces cut perpendicular and parallel to the axis, to make some determinations with an artificial surface parallel to a natural face of the crystal when the crystal was immersed in water; this was accordingly done.

Contrary to the orders that had been given, the edges of the plate were cut away by the optician who polished it, and it was therefore impossible to determine the position of the principal section in the same manner as that previously employed with the other crystals. The results were therefore plotted, the divisions of the ring carrying the crystal being taken as abscissæ and the azimuths of the plane of polarisation of the reflected light as ordinates. It was assumed that the curve for the artificial face would be similar to that for the natural face, and the points at which the curve cut the x axis were taken as indicating the position of the principal section, and the azimuths of the crystal thus determined. Table VIII gives the results :—

Table VIII.

Artificial Surface of Iceland Spar in Water.

Azimuth of the principal section of the crystal.	Azimuth of the plane of polarisation.	Angle of polarisation.	Azimuth of the principal section of the crystal.	Azimuth of the plane of polarisation.	Angle of polarisation.	Difference in the values of the polarising angle at θ and $\theta+180°$.
5°	+0° 32′	46° 47′	185°	+0° 57′	47° 0′	−13′
15	+1 37	46 35	195	+2 42	46 40	− 5
25	+2 12	47 37	205	+4 55	47 12	+25
35	+3 05	47 10	215	+6 40	47 36	−26
45	+3 0	48 48	225	+7 25	48 26	+22
55	+3 02	48 47	235	+8 45	48 50	− 3
65	+0 47	50 13	245	+9 27	49 59	+14
75	+0 02	50 11	255	+8 55	50 06	+ 5
85	−3 33	51 37	265	+7 45	51 06	+31
95	−4 13	50 38	275	+4 10	50 46	− 8
105	−6 38	50 26	285	+2 17	50 39	−13
115	−7 20	49 44	295	+1 22	50 37	−53
125	−8 0	49 53	305	−0 35	50 05	−12
135	−7 10	48 37	315	−1 28	48 55	−18
145	−7 15	48 41	325	−2 58	48 34	+ 7
155	−4 38	47 41	335	−2 08	47 07	+34
165	−3 38	47 23	345	−1 35	47 13	+10
175	−1 30	46 21	355	−0 08	46 02	+19

These results differ considerably from those obtained previously with a natural face in water (Tables II, IV, and VI), and it therefore did not appear worth while to make any further experiments with artificial surfaces, as it seemed certain that the results would be untrustworthy.

The difference between the results obtained with this artificial surface and with a natural surface of the crystal is too great to be explained by supposing that the artificial surface was not cut absolutely parallel to the direction of the cleavage, and must therefore be attributed to some change produced by the polishing, possibly due to the pressure employed (conf. Seebeck, "Pogg. Ann.," vol. xx, 1830, 27).

The values of the azimuths and angles of polarisation given in Tables I, II, III, IV, V, VI, VII, and VIII, were plotted on sectional paper; the azimuths of the principal section of the crystal being taken as abscissæ, and the azimuths of the Nicol, and the angles of polarisation $-40°$, as the ordinates for the two sets of curves.

In order to draw the smooth curves, a piece of plate-glass, rather smaller than the drawing paper, was mounted in a soft-wood frame, so that one surface of the glass was flush with the wood, the sectional paper upon which the observations had been plotted was fixed to the wood with drawing pins, and a sheet of ordinary drawing paper placed over it, and fastened in the same manner. This glass drawing board was then placed in front of a lamp, and smooth curves drawn by eye in the ordinary manner.

Professor Stokes pointed out to me that the experimental results which had been obtained were well suited for reduction by means of the harmonic analysis, and not only explained the method but himself reduced the first set of observations made with a cleavage-face in water. All the observations were accordingly reduced by this method; the determinations made at azimuths 1°, 11°, &c., and at 7° 20′, 17° 20′, &c., in the one series, and at 0°, 10°, &c., and at 5°, 15°, &c., in the second series, being kept separate.

Owing to the fact that the principal section of the crystal is a plane of symmetry, the periodic series for the development of the azimuths of the planes of polarisation can contain sines only, and that for the polarising angles cosines only, including the constant term; therefore the coefficients of the cosines in the former case, and of the sines in the latter, were not calculated, except with the observations made with the artificial surface; it seemed possible that the process of polishing might occasion some want of symmetry, and that therefore it was desirable to calculate the values of the coefficients for both sines and cosines.

The observations in one set only having started at zero azimuth, in the other three there was a small correction to be made in the coefficients for the error thus produced; this was done by multiplying

the coefficients by the secants of 1°, 2°, 3°, 4°, for the orders 1, 2, 3, and 4, in the first set; by the secants of 2° 40′, 5° 20′, 8°, and 10° 40′ in the second, and by the secants of 5°, 10°, 15°, and 20° in the fourth set. This correction only exceeded 1′ in sixteen cases, and attained its maximum value in the second series of observations made in carbon tetrachloride, where in the case of the coefficients of $\sin 2\theta$, $\sin 3\theta$, and $\cos 2\theta$ it amounted to 9′ and 8′ respectively.

Table IX gives the results and their means.

Table IX.

Iceland Spar in Air. Azimuths.

Series I.

Obs. at 1°, 10°, &c.	$23' - 2^\circ\ 14' \sin\theta + 1^\circ\ 47' \sin 2\theta \ldots\ldots\ldots + 4' \sin 4\theta.$
„ −2° 40′, 7° 20′, &c.	$21 - 2\ 11 \sin\theta + 1\ 48 \sin 2\theta + 1' \sin 3\theta - 2 \sin 4\theta.$

Series II.

„ 0°, 10°, &c.	$20 - 2\ 02 \sin\theta + 1\ 51 \sin 2\theta \ldots\ldots\ldots + 5 \sin 4\theta.$
„ 5°, 15°, &c.	$31 - 2\ 12 \sin\theta + 1\ 52 \sin 2\theta + 7' \sin 3\theta - 4 \sin 4\theta.$
Mean (A)	$24 - 2\ 10 \sin\theta + 1\ 49 \sin 2\theta + 2' \sin 3\theta + 1 \sin 4\theta.$

Iceland Spar in Air. Polarising Angles.

Series I.

Obs. at 1°, 10°, &c.	$58^\circ\ 25' - 2' \cos\theta - 1^\circ\ 12' \cos 2\theta + 1' \cos 3\theta + 1' \cos 4\theta.$
„ −2° 40′, 7° 20′, &c.	$58\ 25 + 7 \cos\theta - 1\ 10 \cos 2\theta + 1 \cos 3\theta - 1 \cos 4\theta.$

Series II.

„ 0°, 10°, &c.	$58\ 21 + 14 \cos\theta - 1\ 20 \cos 2\theta + 1 \cos 3\theta + 7 \cos 4\theta.$
„ 5°, 15°, &c.	$57\ 56 + 13 \cos\theta - 1\ 20 \cos 2\theta + 5 \cos 3\theta + 1 \cos 4\theta.$
Mean (B)	$58\ 17 + 8 \cos\theta - 1\ 15 \cos 2\theta + 2 \cos 3\theta + 2 \cos 4\theta.$

Iceland Spar in Water. Azimuths.

Series I.

Obs. at 1°, 10°, &c.	$26' - 9^\circ\ 47' \sin\theta + 5^\circ\ 31' \sin 2\theta + 45' \sin 3\theta - 14' \sin 4\theta.$
„ −2° 40′, 7° 20′, &c.	$23 - 9\ 42 \sin\theta + 5\ 48 \sin 2\theta + 59 \sin 3\theta - 1 \sin 4\theta.$

Series II.

„ 0°, 10°, &c.	$25 - 9\ 11 \sin\theta + 5\ 17 \sin 2\theta + 55 \sin 3\theta - 8 \sin 4\theta.$
„ 5°, 15°, &c.	$41 - 8\ 55 \sin\theta + 5\ 15 \sin 2\theta + 38 \sin 3\theta - 4 \sin 4\theta.$

Series III.

„ 1°, 11°, &c.	$23 - 9\ 39 \sin\theta + 5\ 36 \sin 2\theta + 41 \sin 3\theta - 22 \sin 4\theta.$
Mean (C)	$27 - 9\ 27 \sin\theta + 5\ 29 \sin 2\theta + 47 \sin 3\theta - 10 \sin 4\theta.$

Iceland Spar in Water. Polarising Angles.

Series I.

Obs. at 1°, 10°, &c.	$52^\circ\ 21' + 16' \cos\theta - 3^\circ\ 23' \cos 2\theta + 3' \cos 3\theta + 17' \cos 4\theta.$
„ −2° 40′, 7° 20′, &c.	$52\ 34 + 9 \cos\theta - 3\ 08 \cos 2\theta - 1 \cos 3\theta + 10 \cos 4\theta.$

SERIES II.

Obs. at 0°, 10°, &c. .... $51^{\circ}\ 40' + \ 5' \cos\theta - 3^{\circ}\ 11' \cos 2\theta + 4' \cos 3\theta - 17' \cos 4\theta$.
„ 5°, 15°, &c. .... $51\ 32 + \ 7 \cos\theta - 3\ 12 \cos 2\theta - 7 \cos 3\theta + 10 \cos 4\theta$.

SERIES III.

„ 1°, 11°, &c. .... $52\ 05 - 11 \cos\theta - 3\ 16 \cos 2\theta - 5 \cos 3\theta + 13 \cos 4\theta$.

Mean (D) .... $52\ 02 - \ 5 \cos\theta - 3\ 14 \cos 2\theta - 1 \cos 3\theta + 13 \cos 4\theta$.

Iceland Spar in Tetrachloride of Carbon. Azimuths.

SERIES I.

Obs. at 1°, 11°, &c. .... $39' - 25^{\circ}\ 03' \sin\theta + 11^{\circ}\ 16' \sin 2\theta + 4^{\circ}\ 28' \sin 3\theta + 11' \sin 4\theta$.
„ $-2^{\circ}\ 40'$, $7^{\circ}\ 20'$, &c. $50 - 23\ 31 \sin\theta + 10\ 11 \sin 2\theta + 4\ \ 0 \sin 3\theta - 44 \sin 4\theta$.

SERIES II.

„ 0°, 10°, &c. .... $56 - 23\ 26 \sin\theta + 10\ 17 \sin 2\theta + 4\ 18 \sin 3\theta - 49 \sin 4\theta$.
„ 5°, 15°, &c. .... $28 - 23\ 07 \sin\theta + \ 9\ 55 \sin 2\theta + 4\ 23 \sin 3\theta - 12 \sin 4\theta$.

Mean (E) .... $43 - 23\ 47 \sin\theta + 10\ 25 \sin 2\theta + 4\ 17 \sin 3\theta - 24 \sin 4\theta$.

Iceland Spar in Tetrachloride of Carbon. Polarising Angles.

SERIES I.

Obs. at 1°, 11°, &c. .... $54^{\circ}\ 05' - 17' \cos\theta - 9^{\circ}\ 40' \cos 2\theta + 52' \cos 3\theta + 1^{\circ}\ 18' \cos 4\theta$.
„ $-2^{\circ}\ 40'$, $7^{\circ}\ 20'$, &c. $53\ 01 + \ 4 \cos\theta - 8\ 45 \cos 2\theta + 36 \cos 3\theta + \ \ 59 \cos 4\theta$.

SERIES II.

„ 0°, 10°, &c. .... $52\ 50 + 12 \cos\theta - 8\ 41 \cos 2\theta + 12 \cos 3\theta + 1\ 23 \cos 4\theta$.
„ 5°, 15°, &c. .... $52\ 41 - \ 3 \cos\theta - 8\ 29 \cos 2\theta + \ 7 \cos 3\theta + 1\ 10 \cos 4\theta$.

Mean (F) .... $53\ 09 - \ 1 \cos\theta - 8\ 54 \cos 2\theta + 27 \cos 3\theta + 1\ 12 \cos 4\theta$.

Artificial Surface of Iceland Spar in Water. Azimuths.

(G.) .... $28' - 3^{\circ}\ 52' \sin\theta - 9' \cos\theta + 5^{\circ}\ 11' \sin 2\theta + 11' \cos 2\theta + 33' \sin 3\theta + 7' \cos 3\theta - 21' \sin 4\theta - 4 \cos 4\theta$.

Artificial Surface of Iceland Spar in Water. Polarising Angles.

(H.) ...... $48^{\circ}\ 53' - 1' \sin\theta - 1' \cos\theta + 4' \sin 2\theta - 2^{\circ}\ 09' \cos 2\theta + 4' \sin 3\theta - 8' \cos 3\theta$ ...... $+ 1 \cos 4\theta$.

Brewster, in his paper in the "Phil. Trans." for 1819, p. 158, says, "in any given surface when A and A″ are the maximum and minimum polarising angles, viz., in the azimuths of 0° and 90°, the polarising angle A′ at any intermediate aximuth α, may be found by the formula $A' = A + \sin^2\alpha (A'' - A)$."

This expression is the same as that given by the harmonic reduction of the observations set forth in the preceding pages, if we assume that the smaller terms are due to errors of observation, as in that case the expression for the polarising angle in air (B) becomes $58^{\circ}\ 17' - 1^{\circ}\ 15' \cos 2\theta$.

Calling the coefficient of $\cos 2\theta$ x, and the minimum value of the polarising angle A, this is $(A+x)-x\cos 2\theta$, which is identical with Brewster's expression, since $A''-A$ is the same as $2x$.

Brewster states that for a rhomboidal surface of calcareous spar $A''-A$ 138′, whereas the harmonic reduction gives the value as 150′, which perhaps, considering the nature of the determinations, is as close an agreement as could be well expected.

Brewster's formula also appears to hold good for the case of Iceland spar in water, as the harmonic series for the value of the polarising angle (D) may be taken as $52°\ 02'-3°\ 14'\cos 2\theta$. But with the spar in tetrachloride of carbon the agreement no longer holds, as the coefficient of $\cos 4\theta$ becomes too large to be neglected, being $1°\ 12'$. The determinations made in this strongly refracting liquid were less satisfactory than the others, as is shown by the figures in Tables III and VII, but there is hardly sufficient ground for assuming that the value of the coefficient of $\cos 4\theta$ is merely due to errors of observation.

The experiments of which an account had been given confirm the accuracy of Brewster's observations made with a surface of Iceland spar in contact with media other than air, and show moreover that, as Seebeck pointed out, the change in the value of the azimuth of the plane of polarisation of the reflected light also occurs, though to a far less extent, when the crystal is in air, and further, as the refractive index of the medium increases, the change in both these values is greatly augmented.

The harmonic analysis affords a means of expressing, approximately at least, both these changes as functions of the azimuth of the principal section of the crystal, and further shows that when the crystal is in air or water, Brewster's formula for the angle of polarisation expresses the facts of the case.

The constant term in the expression for the azimuth of the plane of polarisation of the reflected light being due partly to errors of observation and partly to the index error of the Nicol, and, for the reason stated by Professor Stokes in the note he has done me the honour of appending to this paper, the coefficients of the cosines of odd multiples of θ in the expressions for the angles of polarisation being probably due to inaccuracies in the determination, it seems best to omit these terms (which at any rate are extremely small), so that we obtain as the final result the following approximate expressions in the several cases.

Azimuths of the Plane of Polarisation of Light Polarised by Reflection.

Cleavage surf. in air.	$-\ 2°\ 10'\sin\theta+\ 1°\ 49'\sin 2\theta+0°\ \ 2'\sin 3\theta+0°\ \ 1'\sin 4\theta$.
Ditto, in water	$-\ 9°\ 27'\sin\theta+\ 5°\ 29'\sin 2\theta+0°\ 47'\sin 3\theta-0°\ 10'\sin 4\theta$.
Ditto, in CCl_4......	$-23°\ 47'\sin\theta+10°\ 25'\sin 2\theta+4°\ 17'\sin 3\theta-0°\ 24'\sin 4\theta$.
Artificial surf. in water	$-\ 3°\ 52'\sin\theta+\ 5°\ 11'\sin 2\theta+0°\ 33'\sin 3\theta-0°\ 21'\sin 4\theta$.

Polarising Angles.

Cleavage surface in air	$58° \ 17' - 1° \ 15' \cos 2\theta + 0° \ \ 2' \cos 4\theta$.
Ditto, in water	$52° \ \ 2' - 3° \ 14' \cos 2\theta + 0° \ 13' \cos 4\theta$.
Ditto, in CCl_4	$53° \ \ 9' - 8° \ 54' \cos 2\theta + 1° \ 12' \cos 4\theta$.
Artificial surface in water........	$48° \ 53' - 2° \ \ 9' \cos 2\theta + 0° \ \ 1' \cos 4\theta$.

From these expressions the values of the ordinates of the curves representing the phenomena were calculated, and Plates I and II give the curves as plotted from the values so obtained.

These curves correspond very closely with the smooth curves drawn from the points given by the observations, the values of the ordinates for those portions of the curve corresponding to azimuths 0—40°, and 320—360°, being rather greater than the values given by the smooth eye-drawn curve. The curves for the artificial surface in water (G and H) show clearly, when compared with the corresponding curves for the natural surface (C and D), how greatly these two surfaces differed in their optical behaviour.

In conclusion I must express my thanks to Professor Stokes for his advice and assistance, and for all the trouble he has taken with reference to the determinations of which an account is given in this paper.

Note by Professor STOKES, P.R.S.

On inspecting these numbers we may remark :—

1. The coefficients of $\sin 4\theta$ in the expressions for the azimuths are in all cases so small that they can hardly be said to emerge from errors of observation. Since, however, there is no reason to suppose that such a term does not exist, the coefficients may as well be retained, as being somewhat more probable than zero would be.

2. Brewster found that the polarising angles were the same for any two azimuths differing by 180°, and MacCullagh afterwards deduced this result from theoretical considerations. If we assume this law as exact, the harmonic expression for the polarising angle will contain no terms involving cosines of odd multiples of θ. Now with one doubtful exception the coefficients in the above expressions are insensibly small. The single exception, where a coefficient has at first sight the appearance of being real though small, is that of the term involving $\cos 3\theta$ for the observations in tetrachloride of carbon. The observations with this liquid were the most uncertain, probably from the feebleness of reflection arising from its high refractive index. If the differences of the polarising angles for azimuths of the principal plane differing by 180° be examined, it will be seen that a coefficient amounting in the mean to only 0° 27′, and subject to a mean error from set to set of 17′, can have little claim to be regarded as real.

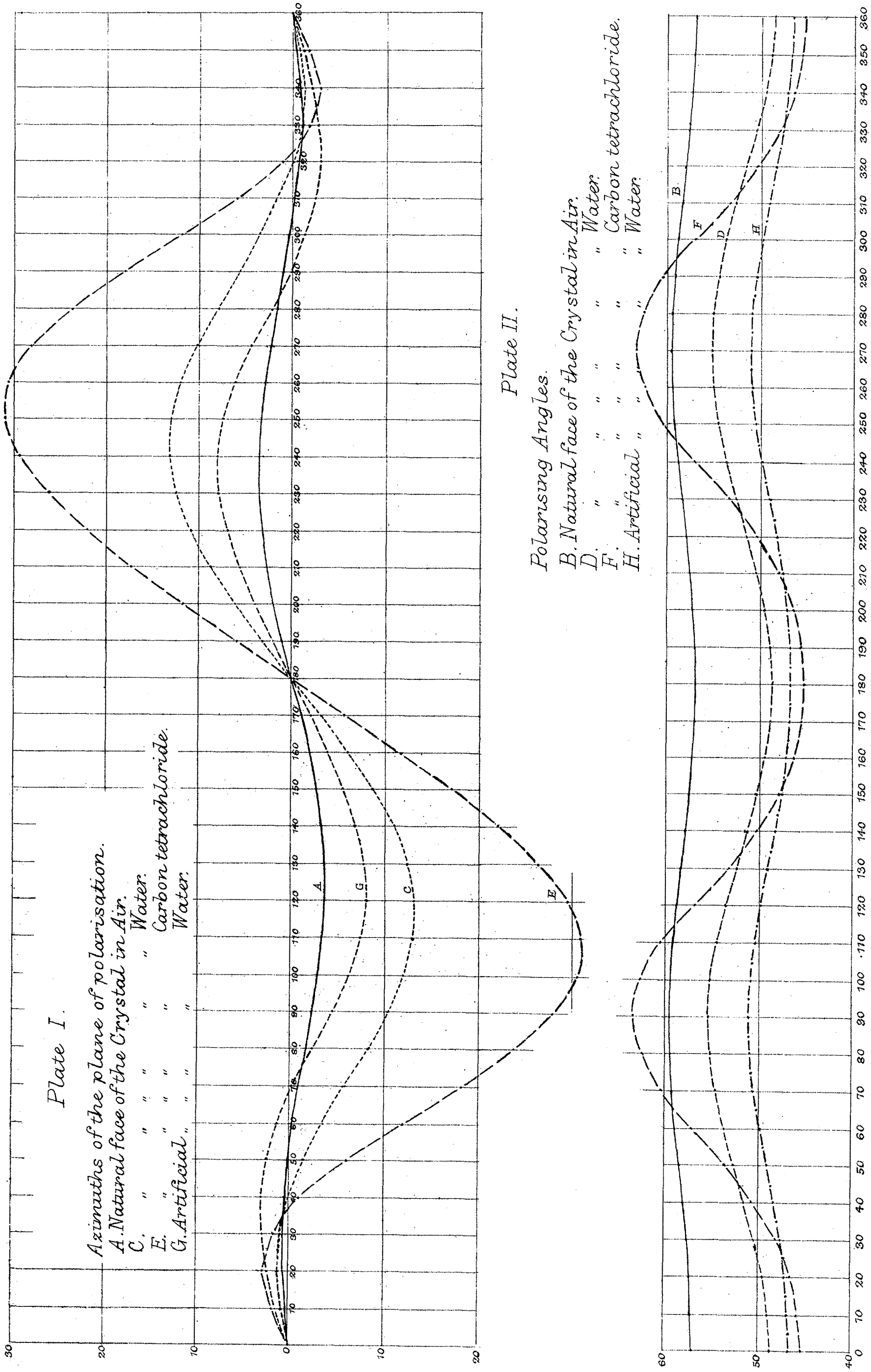

Plate I.

Azimuths of the plane of polarisation.
A. Natural face of the Crystal in Air.
C. " " " " " Water.
E. " " " " " Carbon tetrachloride.
G. Artificial " " " " Water.

Plate II.

Polarising Angles.
B. Natural face of the Crystal in Air.
D. " " " " " Water.
F. " " " " " Carbon tetrachloride.
H. Artificial " " " " Water.

West, Newman & Co. lith.

It seems best therefore in this, as well as other cases, to omit the terms involving cosines of odd multiples of θ.

3. As regards the observations with the artificial surface in water, the coefficients of the cosines in the expression for the azimuths and of the sines in the expression for the polarising angles are insensibly small, indicating no introduction of asymmetry with respect to the principal plane arising from the process of polishing. The coefficients of the cosines of odd multiples of θ in the second expression are also insensible. The constant term in the first expression, representing (on the assumption of symmetry with respect to the principal plane) the index error of the circle carrying the Nicol, agrees almost exactly with those obtained for the cleavage surfaces in air and water.

It would seem best then to omit those terms which we have reason to think are really *nil*, and which the observations show to be at any rate extremely small, and to exhibit the final result accordingly.

February 11, 1886.

Professor STOKES, D.C.L., President, in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read:—

I. "On the Theory of Lubrication and its Application to Mr. Beauchamp Tower's Experiments, including an Experimental Determination of the Viscosity of Olive Oil." By Professor OSBORNE REYNOLDS, LL.D., F.R.S. Received December 29, 1885.

(Abstract.)

Lubrication, or the action of oils and other viscous fluids to diminish friction and wear between solid surfaces, does not appear to have hitherto formed a subject for theoretical treatment. Such treatment may have been prevented by the obscurity of the physical actions involved, which belong to a class as yet but little known, namely, the boundary or surface actions of fluids; but the absence of such treatment has also been owing to the want of any general laws revealed by experiment.

The subject is of such fundamental importance in practical mechanics, and the opportunities of observation so frequent, that it